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Vacuum-Mimicking Behavior in Neutrino Oscillation Dynamics

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Abstract

It is shown generally that any oscillation probability in matter with approximately constant density coincides with that in vacuum to the first two nontrivial orders in m2 JKL/E if |m2 JKL/E| 1 and |GF Nell| 1 are satisfied. □ 2001 Published by Elsevier Science B.V

Recently a lot of efforts have been made on study of neutrino oscillations at long baseline experiments. Using the mass hierarchical condition |m2 21||m2 32|————————————|m2 31| in the three-flavour framework of neutrino oscillations, it has been found in the case of T-conserving probability P (he → νμ) [1–3] or in the case of T-violating probability P (νμ → he) [4,5] that the oscillation probability P (να → νβ) matter in matter coincides with that P (να → νβ) vacuum in vacuum

$$P(\nu_\alpha \to \nu_\beta)_{\mathrm{matter}} \simeq P(\nu_\alpha \to \nu_\beta)_{\mathrm{vacuum}},$$

when |m2 JKL/E| 1 and |AL| 1 are satisfied, where A ≡ √2 GF Ne stands for the matter effect [6,7] and Ne is the density of electrons. This phenomenon was referred to as vacuum mimicking in [5]. In this short note it is shown that (1) holds in the first two nontrivial orders in m2 JKL/2E and AL (the terms quadratic and cubic in m2 JKL/2E correspond to T-conserving and T-violating probabilities in the leading order, respectively) for arbitrary numbers N of neutrino flavours with general form dig (A1, A2, AN) of the matter effect if |mukluk/2E| 1 and |AL| 1 are satisfied. In the three-flavour framework of neutrino oscillations, the positive energy part of the Dirac equation which describes neutrino propagation is given by

$$i\frac{d\Psi}{dt} = \left[U\,\mathrm{diag}(E_1, E_2, E_3)U^{-1} + \mathrm{diag}(A, 0, 0)\right]\Psi,$$

where ΨT ≡ (he, νμ, ντ) is the flavour eigenstate, U is the Pontecorvo–Maki–Nakagawa–Sakata [8–10] (PMNS) matrix, 1 and Edi ≡ m2 j + p2. Throughout this paper we assume that the density of matter is constant for simplicity. The case of matter with slowly varying density will be briefly discussed at the end of the Letter. Here let us consider more general case with N neutrino flavours and with general matter effect:

$$i\frac{d\Psi}{dt} = \left(U\mathcal{E}U^{-1} + A\right)\Psi, \qquad (3)$$

where

$$\mathcal{E} \equiv \mathrm{diag}(E_1, E_2, \ldots, E_N), \qquad (4)$$

$$A \equiv \mathrm{diag}(A_1, A_2, \ldots, A_N), \qquad (5)$$

U is the $N \times N$ PMNS matrix and $\Psi^T \equiv (\nu_{\alpha_1}, \nu_{\alpha_2}, \ldots, \nu_{\alpha_N})$ is the flavor eigenstate. Without the matter effect (i.e., $A_j = 0, j = 1, \ldots, N$), (3) can be easily solved and the oscillation probability $P(\nu_\alpha \to \nu_\beta)_{\mathrm{vacuum}}$ is given by

$$P(\nu_\alpha \to \nu_\beta)_{\mathrm{vacuum}} = \delta_{\alpha\beta} - 2\sum_{j,k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\sin^2\left(\frac{\Delta E_{jk}L}{2}\right) - i\sum_{j,k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\sin(\Delta E_{jk}L), \qquad (6)$$

where Elk ≡ Edi − Ek and the second and the third terms on the right-hand side correspond to CP-conserving and CP-violating probabilities, respectively. With the nonvanishing matter effect, on the other hand, explicit evaluation of the probability is difficult but the N × N matrix UEU−1 + A on the right-hand side of (3) can be formally diagonalized by an N × N unitary matrix UM:

$$U\mathcal{E}U^{-1} + A = U^M\mathcal{E}^M\left(U^M\right)^{-1}, \qquad (7)$$

where

$$\mathcal{E}^M \equiv \mathrm{diag}\left(E^M_1, E^M_2, \ldots, E^M_N\right), \qquad (8)$$

and E^M_j stands for the eigenvalue of $U\mathcal{E}U^{-1} + A$. As in the case of the oscillation probability in vacuum, we can formally solve (3) and express the oscillation probability $P(\nu_\alpha \to \nu_\beta)_{\mathrm{matter}}$ as

$$P(\nu_\alpha \to \nu_\beta)_{\mathrm{matter}} = \delta_{\alpha\beta} - 2\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\sin^2\left(\frac{\Delta E^M_{jk}L}{2}\right) - i\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\sin\left(\Delta E^M_{jk}L\right), \qquad (9)$$

where EM joke ≡ EM j − EM k and the second and the third terms on the right-hand side correspond to T-conserving and T-violating probabilities, respectively. Now let us assume that |JKL| 1 and |EM JKL| 1 are satisfied, where the latter follows if |JKL| 1 and |AL| 1. Then we can expand the sine functions in (6) and (9). The zeroth order term is obviously δαβ for both probabilities. The term linear in JKL vanishes, since

$$\sum_{j,k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\Delta E_{jk}L = L\sum_{j,k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}(E_j - E_k)$$

$$= L\left[\delta_{\alpha\beta}\sum_j U_{\alpha j}U^*_{\beta j}E_j - \delta_{\alpha\beta}\sum_k U^*_{\alpha k}U_{\beta k}E_k\right] = L\delta_{\alpha\beta}\left[\left(U\mathcal{E}U^{-1}\right)_{\alpha\beta} - \left(U\mathcal{E}U^{-1}\right)_{\beta\alpha}\right] = 0,$$

where δαβ has been obtained from the unitarity condition j Uαj U∗ βj = δαβ, and the last equality holds because the inside of the square bracket vanishes for α = β. Similarly, we have

$$\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\Delta E^M_{jk}L = L\delta_{\alpha\beta}\left[\left(U^M\mathcal{E}(U^M)^{-1}\right)_{\alpha\beta} - \left(U^M\mathcal{E}(U^M)^{-1}\right)_{\beta\alpha}\right] = 0. \quad (11)$$

The first nontrivial case is the term quadratic in $\Delta E_{jk}L$ and $\Delta E^M_{jk}L$. From (9) we have the term quadratic in $\Delta E^M_{jk}L$ (up to a factor $-1/2$)

$$\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\left(\Delta E^M_{jk}L\right)^2 = L^2\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\left[\left(E^M_j\right)^2 - 2E^M_jE^M_k + \left(E^M_k\right)^2\right]$$

$$= L^2\left[\delta_{\alpha\beta}\sum_j U^M_{\alpha j}U^{M*}_{\beta j}\left(E^M_j\right)^2 + \delta_{\alpha\beta}\sum_k U^{M*}_{\alpha k}U^M_{\beta k}\left(E^M_k\right)^2 - 2\sum_j U^M_{\alpha j}U^{M*}_{\beta j}E^M_j\sum_k U^{M*}_{\alpha k}U^M_{\beta k}E^M_k\right]. \quad (12)$$

Here we note the following properties:

$$\sum_j U^M_{\alpha j}U^{M*}_{\beta j}E^M_j = \left(U\mathcal{E}U^{-1} + A\right)_{\alpha\beta} = \left(U\mathcal{E}U^{-1}\right)_{\alpha\beta} + \delta_{\alpha\beta}A_\alpha, \quad (13)$$

$$\sum_j U^M_{\alpha j}U^{M*}_{\beta j}\left(E^M_j\right)^2 = \left[U^M\left(\mathcal{E}^M\right)^2\left(U^M\right)^{-1}\right]_{\alpha\beta} = \left\{\left[U^M\mathcal{E}^M\left(U^M\right)^{-1}\right]^2\right\}_{\alpha\beta} = \left[\left(U\mathcal{E}U^{-1} + A\right)^2\right]_{\alpha\beta}$$

$$= \left(U\mathcal{E}^2U^{-1}\right)_{\alpha\beta} + \left(A_\alpha + A_\beta\right)\left(U\mathcal{E}U^{-1}\right)_{\alpha\beta} + \delta_{\alpha\beta}\left(A_\alpha\right)^2. \quad (14)$$

Thus (12) becomes

$$L^2\delta_{\alpha\beta}\left\{\left[U^M\left(\mathcal{E}^M\right)^2\left(U^M\right)^{-1}\right]_{\alpha\beta} + \left[U^M\left(\mathcal{E}^M\right)^2\left(U^M\right)^{-1}\right]_{\beta\alpha}\right\}$$

$$-2L^2\left[U^M\mathcal{E}^M\left(U^M\right)^{-1}\right]_{\alpha\beta}\left[U^M\mathcal{E}^M\left(U^M\right)^{-1}\right]_{\beta\alpha}$$

$$= 2L^2\delta_{\alpha\beta}\left[\left(U\mathcal{E}^2U^{-1}\right)_{\alpha\alpha} + 2A_\alpha\left(U\mathcal{E}U^{-1}\right)_{\alpha\alpha} + \left(A_\alpha\right)^2\right]$$

$$-2L^2\left[\left(U\mathcal{E}U^{-1}\right)_{\alpha\beta} + \delta_{\alpha\beta}A_\alpha\right]\left[\left(U\mathcal{E}U^{-1}\right)_{\beta\alpha} + \delta_{\alpha\beta}A_\alpha\right]$$

$$= 2L^2\left[\delta_{\alpha\beta}\left(U\mathcal{E}^2U^{-1}\right)_{\alpha\alpha} - \left(U\mathcal{E}U^{-1}\right)_{\alpha\beta}\left(U\mathcal{E}U^{-1}\right)_{\beta\alpha}\right], \quad (15)$$

where all the contributions of the matter effect have disappeared in the last step. Since the last expression in (15) is the term quadratic in JKL for the probability in vacuum, we obtain

$$\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\left(\Delta E^M_{jk}L\right)^2 = \sum_{j,k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\left(\Delta E_{jk}L\right)^2.$$

Next let us turn to the term cubic in $\Delta E^M_{jk}L$. It is given by (up to a factor $i/3!$)

$$\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\left(\Delta E^M_{jk}L\right)^3$$

$$= L^3\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\left[\left(E^M_j\right)^3 - 3\left(E^M_j\right)^2E^M_k + 3E^M_j\left(E^M_k\right)^2 - \left(E^M_k\right)^3\right]$$

$$= L^3\delta_{\alpha\beta}\left\{\left[U^M\left(\mathcal{E}^M\right)^3\left(U^M\right)^{-1}\right]_{\alpha\beta} - \left[U^M\left(\mathcal{E}^M\right)^3\left(U^M\right)^{-1}\right]_{\beta\alpha}\right\}$$

$$-3L^3\left[U^M\left(\mathcal{E}^M\right)^2\left(U^M\right)^{-1}\right]_{\alpha\beta}\left[U^M\mathcal{E}^M\left(U^M\right)^{-1}\right]_{\beta\alpha}$$

$$+3L^3\left[U^M\mathcal{E}^M\left(U^M\right)^{-1}\right]_{\alpha\beta}\left[U^M\left(\mathcal{E}^M\right)^2\left(U^M\right)^{-1}\right]_{\beta\alpha}$$

$$= -3L^3\left[\left(U\mathcal{E}^2U^{-1}\right)_{\alpha\beta} + \left(A_\alpha + A_\beta\right)\left(U\mathcal{E}U^{-1}\right)_{\alpha\beta} + \delta_{\alpha\beta}\left(A_\alpha\right)^2\right]\left[\left(U\mathcal{E}U^{-1}\right)_{\beta\alpha} + \delta_{\alpha\beta}A_\alpha\right]$$

$$+3L^3\left[\left(U\mathcal{E}U^{-1}\right)_{\alpha\beta} + \delta_{\alpha\beta}A_\alpha\right]\left[\left(U\mathcal{E}^2U^{-1}\right)_{\beta\alpha} + \left(A_\alpha + A_\beta\right)\left(U\mathcal{E}U^{-1}\right)_{\beta\alpha} + \delta_{\alpha\beta}\left(A_\alpha\right)^2\right]$$

$$= -3L^3\left[\left(U\mathcal{E}^2U^{-1}\right)_{\alpha\beta}\left(U\mathcal{E}U^{-1}\right)_{\beta\alpha} - \left(U\mathcal{E}U^{-1}\right)_{\alpha\beta}\left(U\mathcal{E}^2U^{-1}\right)_{\beta\alpha}\right],$$

where all the contributions of the matter effect have disappeared again in the last step. Since the last expression in (17) is the term cubic in JKL for the probability in vacuum, we obtain

It turns out that the matter contributions in the terms of $O((\Delta E_{jk}L)^4)$ or higher are not canceled and we have

$$P(\nu_\alpha \to \nu_\beta)_{\text{matter}} = P(\nu_\alpha \to \nu_\beta)_{\text{vacuum}} + O\left((\Delta E_{jk}L)^4\right). \quad (19)$$

We note in passing that Eq. (18) gives another proof of the Harrison–Scott identity [12] for the case with three flavors [2]

$$J^M\Delta E^M_{31}\Delta E^M_{32}\Delta E^M_{21} = J\Delta E_{31}\Delta E_{32}\Delta E_{21}, \quad (20)$$

for

$$\sum_{j,k} U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\left(\Delta E^M_{jk}\right)^3$$

$$= i\sum_{j<k} \Im\left(U^M_{\alpha j}U^{M*}_{\beta j}U^{M*}_{\alpha k}U^M_{\beta k}\right)\left(\Delta E^M_{jk}\right)^3 = iJ^M\left[-\left(\Delta E^M_{13}\right)^3 + \left(\Delta E^M_{23}\right)^3 + \left(\Delta E^M_{12}\right)^3\right]$$

$$= -3iJ^M\Delta E^M_{31}\Delta E^M_{32}\Delta E^M_{21} = \sum_{j,k} U_{\alpha j}U^*_{\beta j}U^*_{\alpha k}U_{\beta k}\left(\Delta E_{jk}\right)^3 = -3iJ\Delta E_{31}\Delta E_{32}\Delta E_{21}, \quad (21)$$

where

$$J^M \equiv \Im\left(U^M_{\alpha 1}U^{M*}_{\beta 1}U^{M*}_{\alpha 2}U^M_{\beta 2}\right), \quad (22)$$

$$J \equiv \Im\left(U_{\alpha 1}U^*_{\beta 1}U^*_{\alpha 2}U_{\beta 2}\right) \quad (23)$$

3 + b3 − (a + b)3 = −3ab (a + b) = 3abc for a + b + c = 0 (a ≡ E13, b ≡ E32, c ≡ E21). For long baseline experiments such as JHF [14] with relatively low energy (El ∼ 1 GeV, L ∼ 300 km), the larger mass squared difference |m2 32| ∼ 3 × 10−3 eV2 gives |m2 32L/2E| ∼ O (1) and our assumption does not hold. In fact, it has been shown [15] that there is some contribution from the matter effect to CP violation at the JHF neutrino experiment. So far, we have assumed that the density of matter is approximately constant. However, even if the density depends on the position, if adiabatic treatment is allowed (i.e., |dump/dt||EM j |) then we can apply our argument to each interval in which the density can be regarded as approximately constant. Hence, vacuum mimicking phenomena occur if adiabatic treatment is justified and |JKL| 1 and |AL| 1 are satisfied.

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